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Corrigendum

On the derivative of the Legendre function of the first kind with respect to its degree
Radosław Szmytkowski 2006 *J. Phys. A: Math. Gen.* **39** 15147–15172

Our claim that at $t = z$ the integrand in

$$R_n(z) = \frac{1}{2^n \pi \mathrm{i}} \oint_{\mathcal{C}^{(+)}} \mathrm{d}t \, \frac{\left(t^2-1\right)^n}{(t-z)^{n+1}} \ln \frac{t+1}{z+1} \tag{5.5}$$

has a pole of order $n + 1$ is erroneous. The presence of $\ln[(t+1)/(z+1)]$ (we recall that the principal branch of the logarithm is used) causes that if z is not on the cut $(-\infty, -1]$, the point $t = z$ is regular for $n = 0$ or (when $z \neq 1$) is a pole of order n for $n \geqslant 1$. Thus, under the aforementioned restrictions imposed on z (in fact, these restrictions may be relaxed by carrying out a reasoning analogous to that presented below in the support of equation (5.6), by a direct application of the theory of residues to the integral in equation (5.5), one obtains

$$R_n(z) = \frac{1}{2^{n-1}(n-1)!} \left[\frac{\mathrm{d}^{n-1}}{\mathrm{d}t^{n-1}} \frac{\left(t^2-1\right)^n}{t-z} \ln \frac{t+1}{z+1} \right]_{t=z}.$$

Nevertheless, the mistake we have made does not invalidate any formula in the paper. In particular, the relation

$$R_n(z) = \frac{1}{2^{n-1} n!} \left[\frac{\mathrm{d}^n}{\mathrm{d}t^n} \left(t^2-1\right)^n \ln \frac{t+1}{z+1} \right]_{t=z} \tag{5.6}$$

remains valid despite our reasoning leading to it being incorrect.

The correct argument supporting equation (5.6) is as follows.

Assume at first that $z \neq 1$ and that z is not on the cut $(-\infty, -1]$. Then it is evident that equation (5.5) may be rewritten as

$$R_n(z) = \frac{1}{2^n \pi \mathrm{i}} \oint_{\mathcal{C}^{(+)}} \mathrm{d}t \, \frac{\left(t^2-1\right)^n}{(t-z)^{n+1}} \ln(t+1) - \frac{1}{2^n \pi \mathrm{i}} \oint_{\mathcal{C}^{(+)}} \mathrm{d}t \, \frac{\left(t^2-1\right)^n}{(t-z)^{n+1}} \ln(z+1).$$

Under the assumptions made, at $t = z$ both integrands in the above have poles of order $n + 1$ (except for the case of $z = 0$ when the second integral vanishes) and by the theory of residues one obtains

$$R_n(z) = \frac{1}{2^{n-1} n!} \left[\frac{\mathrm{d}^n}{\mathrm{d}t^n} \left(t^2-1\right)^n \ln(t+1) \right]_{t=z} - \frac{1}{2^{n-1} n!} \left[\frac{\mathrm{d}^n}{\mathrm{d}t^n} \left(t^2-1\right)^n \ln(z+1) \right]_{t=z}.$$

Hence, equation (5.6) follows immediately. It is easy to verify that equation (5.6) also remains valid if $z = 0$. When $z = 1$, the integrand in (5.5) is analytic in the domain enclosed by $\mathcal{C}^{(+)}$, so that it holds that $R_n(1) = 0$. The same result follows from equation (5.6), which is thus proved to be valid for $z \in \mathbb{C} \setminus (-\infty, -1]$.

Next, consider the case $z \in (-\infty, -1)$. We have

$$R_n(x \pm \mathrm{i}0) = \lim_{z \to x \pm \mathrm{i}0} R_n(z) \qquad (-\infty < x < -1).$$

Since from equation (5.6) (proved so far for $z \in \mathbb{C} \setminus (-\infty, -1])$ it may be shown (cf equation (5.9) in the paper) that $R_n(z)$ is a polynomial in z of degree n, the two limits in the above equation must be identical, being equal to the polynomial $R_n(x)$. Thus, one may use equation (5.6) to represent $R_n(z)$ also for $z \in (-\infty, -1)$.

For $z = -1$ we define

$$R_n(-1) = \lim_{z \to -1} R_n(z).$$

Since $R_n(z)$ is the polynomial in z (cf the remark in the preceding paragraph), the limit clearly exists, being the value of this polynomial at $z = -1$.